

Hillside Floor Area Ratio Gradual Formula

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Definitions and Conditions

S = Average Slope, given as a percent,

with the Condition that the maximum for **S** = 50%

i.e., if **S** > 50%, then **S** = 50%

A = Lot Area in square feet,

with the Condition that the maximum for **A** = 4,356(100 - **S**)

i.e., if **A** > 4,356(100 - **S**), then **A** = 4,356(100 - **S**)

F = Floor Area Guideline or Limitation,

with the Condition that **F** is subject to Ross Ordinance 18.41.100, which has a recommended guideline maximum of 10,000 square feet total floor area.

The Frankel Hillside Lot Gradual FAR Formula:

$$F = (0.15 - 0.002S)A - 0.005 (A^2/43,560)$$

Explanation of Conditions for A and for S

1) If $S > 50\%$, $S = 50\%$. The formula only works up to 50% slope, because it is based on smoothing out the previous Ross system of stepped slope percentages, which only went to over 50% (“51% and over”). In this previous system, the allowed floor area as percentage of lot size dropped by 1% in each new stepped section. At over 50%, the previous stepped percentages were 4% of the first acre, 3% of the second acre, and 2% of the third acre. If you kept this system going into higher slopes, you would end up with negative floor allowances. (For example, a lot slope of over 65% would allow minus 1% for the third acre, which is obviously not intended).

2) If $A > 4,356(100 - S)$, then $A = 4,356(100 - S)$. The maximum lot size in square feet (**A**) that may be used in this formula is : 4,356(100-**S**). In other words, the maximum number of acres that can be used in the formula is (100-**S**)/10. For example, at a slope of 30%, the maximum acres would be 70/10 = 7. At a slope of 40%, the maximum acres would be 60/10 = 6. This is because the previous stepped system decreased the allowed floor area by 1% for every additional acre, and at a certain number of acres, this becomes a negative number. For example, at 30%, the first acre is allowed 8%, the 2nd acre 7%, and so on, but the 10th acre would be allowed minus 1%. At 50% slope, the 6th acre would be allowed minus 1%. This system obviously doesn’t work for indefinite number of acres. This condition in the formula works really well, because at lower slopes, you hit the 10,000 square foot limitation before you hit the maximum number of acres, so using more acres in the formula wouldn’t give you more floor area square feet anyway.

There are two parts to this formula, which were combined to make the formula above.

Formula for Decreasing Allowed FAR As Slope Increases

How Much Floor Area Decreases As Slope Increases

To gradually decrease the allowable percent as slope increases, the previous stepped formula decreased the allowed floor area by 1% for every 5% increase in slope over 30%. (For example, for the first acre, at 36-40% slope, 3,049 square feet were allowed, which is 7% of one acre. At 41-45% slope, 2,614 square feet were allowed, which is 6% of one acre.) A 1% decrease in allowed floor area for every 5% slope increase is equivalent to a 0.2% decrease in floor area for every 1% slope increase (because $1/5 = 0.2$).

Where Slope Calculation Starts

The first level of slope for calculations was from 30% up to 36%. The middle of this range is 33%. Therefore, I started this formula at 33%. To replace this stepped system with a smooth formula, you just subtract 33% from the slope, multiply this by 0.002 (decimal equivalent to 0.2%), and then subtract this from the allowed Floor Area Percentage. For example, here's how to make this into a formula for the 8% initial allowed percent for the first acre.

Call P the decimal percent of the area that will be allowed as floor area (the FAR).

$P = 0.08 - (S - 33) \times 0.002$ This is just a restatement of the stepwise system.

Expanding this gives

$$P = 0.08 - 0.002S + 33 \times 0.002 = 0.08 - 0.002S + 0.066$$

$$P = 0.08 + 0.066 - 0.002S$$

$$P = 0.146 - 0.002S$$

Gradual Lot Size Formula

The second part of the Frankel formula at the top is that, instead of stepping the basic allowed percent from 8% to 7% to 6% for 1, 2, and 3 acres respectively, I used the Gradual Lot Size Formula (for a 30% slope only):

$$F = 0.085A - 0.005 (A^2/43,560)$$

The derivation is explained below. This formula gives the exact same number of square feet as the previous stepwise system for exactly 1 acre, 2 acres, or 3 acres, and is extremely close to the stepwise system for in-between sizes.

Combining the Two Formulas

To combine these two formulas, you need to modify the Gradual Lot Size Formula for 30% Slope to account for steeper slopes. To do this, we just modify the first term, 0.085, as follows. Call this term P.

At 33% slope, $P = 0.085$

Modify to account for steeper slopes, exactly the same as we did with 0.08 above:

$P = 0.085 - (\mathbf{S} - 33) \times 0.002$ This is just a restatement of the stepwise system.

Expanding this gives

$P = 0.085 - 0.002\mathbf{S} + 33 \times 0.002 = 0.085 - 0.002\mathbf{S} + 0.066$

$P = 0.085 + 0.066 - 0.002\mathbf{S}$

$P = 0.151 - 0.002\mathbf{S}$

To simplify this, I rounded off the first term of 0.151 to 0.15, giving

$P = 0.15 - 0.002\mathbf{S}$

So we have two formulas to combine:

$F = 0.085\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$

with the 0.085 modified to be $0.15 - 0.002\mathbf{S}$

which gives the final formula:

$F = (0.15 - 0.002\mathbf{S})\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$

Derivation of Gradual Lot Size Formula (for 33% slope) Gradually Decrease FAR as Lot Size Increases

I started with the assumption that for your first square foot, you would get 8.5% (0.085 square feet), and this would gradually decrease by 1% over the first acre, so that the end result would be the same as 8% for the whole first acre. In other words, the decrease would be 1/43,560 of 1% (0.01) for every square foot added.

Suppose your lot is **A** square feet. Your total allowed Floor Area **F** would be:

$$\begin{aligned}
 F &= 0.085 - .01(1/43,560) \\
 &+ 0.085 - .01(2/43,560) \\
 &+ 0.085 - .01(3/43,560) \\
 &\dots \\
 &\dots \\
 &+ 0.085 - .01(A/43,560)
 \end{aligned}$$

Mathematically,

$$F = \sum_1^A 0.085 - 0.01 \frac{\sum_{n=1}^{n=A} n}{43,560}$$

As math students know,

$$\sum_{n=1}^{n=A} n = A(A+1)/2 \quad \text{which I will simplify to } A^2/2$$

Therefore,

$$F = 0.085A - 0.01 (A^2/2) / 43,560$$

which simplifies to:

$$F = 0.085A - 0.005 (A^2/43,560)$$

If this formula is compared with the stepwise formula for various lot sizes, the results will be very close. This gradual formula is more elegant.