

Hillside Floor Area Ratio Gradual Formula

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Definitions:

A = Lot Area in square feet

S = Average Slope, given as a percent

F = Floor Area Allowed

The Frankel Hillside Lot Gradual FAR Formula:

$$F = (0.13 - 0.0015S)A - 0.005 (A^2/43,560)$$

There are two parts to this formula, which were combined to make the formula above.

Formula for Decreasing Allowed FAR As Slope Increases

To gradually decrease the allowable percent as slope increases, the previous stepped formula was to decrease the allowed FAR by 0.15% for every 1% increase in slope over 30%. To replace this with a smooth formula, you just subtract 30% from the slope, multiply this by 0.0015, and then subtract this from the allowed Floor Area Percentage. For example, here's how to make this into a formula for the 8% initial allowed percent for the first acre.

Call P the decimal percent of the area that will be allowed as floor area (the FAR).

$$P = 0.08 - (S - 30) \times 0.0015 \quad \text{This is just a restatement of the stepwise system.}$$

Expanding this gives

$$P = 0.08 - 0.0015S + 30 \times 0.0015 = 0.08 - 0.0015S + 0.045$$

$$P = 0.08 + 0.045 - 0.0015S$$

$$P = 0.125 - 0.0015S$$

Gradual Lot Size Formula

The second part of the Frankel formula at the top is that, instead of stepping the basic allowed percent from 8% to 7% to 6% for 1, 2, and 3 acres respectively, I used the Gradual Lot Size Formula (for a 30% slope only):

$$F = 0.085A - 0.005 (A^2/43,560)$$

The derivation is explained below. This formula gives the exact same number of square feet as the previous stepwise system for exactly 1 acre, 2 acres, or 3 acres, and is extremely close to the stepwise system for in-between sizes.

Combining the Two Formulas

To combine these two formulas, you need to modify the Gradual Lot Size Formula for 30% Slope to account for steeper slopes. To do this, we just modify the first term, 0.085, as follows. Call this term P.

At 30% slope, $P = 0.085$

Modify to account for steeper slopes, exactly the same as we did with 0.08 above:

$P = 0.085 - (S - 30) \times 0.0015$ This is just a restatement of the stepwise system.

Expanding this gives

$$P = 0.085 - 0.0015S + 30 \times 0.0015 = 0.085 - 0.0015S + 0.045$$

$$P = 0.085 + 0.045 - 0.0015S$$

$$P = 0.13 - 0.0015S$$

So we have two formulas to combine:

$$F = 0.085A - 0.005 (A^2/43,560)$$

with the 0.085 modified to be $0.13 - 0.0015S$

which gives the final formula:

$$F = (0.13 - 0.0015S)A - 0.005 (A^2/43,560)$$

Derivation of Gradual Lot Size Formula (for 30% slope)
Gradually Decrease FAR as Lot Size Increases

I started with the assumption that for your first square foot, you would get 8.5% (0.085 square feet), and this would gradually decrease by 1% over the first acre, so that the end result would be the same as 8% for the whole first acre. In other words, the decrease would be 1/43,560 of 1% (0.01) for every square foot added.

Suppose your lot is **A** square feet. Your total allowed Floor Area **F** would be:

$$\begin{aligned}
 F &= 0.085 - .01(1/43,560) \\
 &+ 0.085 - .01(2/43,560) \\
 &+ 0.085 - .01(3/43,560) \\
 &\dots \\
 &\dots \\
 &+ 0.085 - .01(A/43,560)
 \end{aligned}$$

Mathematically,

$$F = \sum_1^A 0.085 - 0.01 \frac{\sum_{n=1}^A n}{43,560}$$

As math students know,

$$\sum_{n=1}^A n = A(A+1)/2 \quad \text{which I will simplify to } A^2/2$$

Therefore,

$$F = 0.085A - 0.01 (A^2/2) / 43,560$$

which simplifies to:

$$F = 0.085A - 0.005 (A^2/43,560)$$

If this formula is compared with the stepwise formula for various lot sizes, the results will be very close, with a very slight benefit to the property owner. This gradual formula is more elegant.

Comparing the Frankel Formula and the Stepwise Formula for 30% Slope

For a 30% slope, the formula simplifies as follows:

The starting formula is:

$$F = (0.13\mathbf{A} - 0.0015\mathbf{S})\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$$

Putting in the 30% slope gives:

$$F = (0.13 - 0.0015(30))\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$$

Multiplying the slope by 0.0015 gives:

$$F = (0.13 - 0.045)\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$$

Combining numbers

$$F = 0.085\mathbf{A} - 0.005 (\mathbf{A}^2/43,560)$$

For 1/2 acre (21,780 sf)

$$\text{Stepwise } F = 0.08(21,780) = \underline{1,742 \text{ sf allowed.}}$$

$$\begin{aligned} \text{Frankel } F &= 0.085(21,780) - 0.005(21,780^2)/43,560 \\ &= 0.085(21,780) - 0.005(0.5)(21,780) \\ &= 0.0825(21,780) = \underline{1,797 \text{ sf allowed.}} \end{aligned}$$

for 1 acre (43,560 sf):

$$\text{Stepwise } F = 0.08(43,560) = \underline{3,485 \text{ sf allowed}}$$

$$\begin{aligned} \text{Frankel } F &= 0.085(43,560) - 0.005(43,560^2)/43,560 \\ &= 0.085(43,560) - 0.005(43,560) \\ &= 0.08(43,560) = \underline{3,485 \text{ sf allowed}} \end{aligned}$$

The two methods give the same result.